

Conceptual tensions between quantum mechanics and general relativity: Are there experimental consequences?

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Abstract

One of the conceptual tensions between quantum mechanics (QM) and general relativity (GR) arises from the clash between the *spatial nonseparability* of entangled states in QM, and the complete *spatial separability* of all physical systems in GR, i.e., between the *nonlocality* implied by the superposition principle, and the *locality* implied by the equivalence principle. Possible experimental consequences of this conceptual tension will be discussed for macroscopically entangled, coherent quantum fluids, such as superconductors, superfluids, atomic Bose-Einstein condensates, and quantum Hall fluids, interacting with tidal and gravitational radiation fields. A minimal-coupling rule, which arises from the electron spin coupled to curved spacetime, leads to an interaction between electromagnetic (EM) and gravitational (GR) radiation fields mediated by a quantum Hall fluid. This suggests the possibility of a quantum transducer action, in which EM waves are convertible to GR waves, and vice versa.

1 Introduction

“Mercy and Truth are met together; Righteousness and Peace
have kissed each other.” (Psalm 85:10)

In this Festschrift Volume in honor of John Archibald Wheeler, I would like to take a fresh look at the intersection between two fields to which he devoted much of his research life: general relativity (GR) and quantum mechanics (QM). As evidence of his keen interest in these two subjects, I would cite two examples

from my own experience. When I was an undergraduate at Princeton University during the years from 1957 to 1961, he was my adviser. One of his duties was to assign me topics for my junior paper and for my senior thesis. For my junior paper, I was assigned the topic: Compare the complementarity and the uncertainty principles of quantum mechanics: Which is more fundamental? For my senior thesis, I was assigned the topic: How to quantize general relativity? As Wheeler taught me, more than half of science is devoted to the asking of the right question, while often less than half is devoted to the obtaining of the correct answer, but not always!

In the same spirit, I would like to offer up here some questions concerning conceptual tensions between GR and QM, which hopefully can be answered in the course of time by experiments, with a view towards probing the tension between the concepts of *locality* in GR and *nonlocality* in QM. I hope that it would be appropriate and permissible to ask some questions here concerning this tension. It is not the purpose of this Chapter to present demonstrated results, but to suggest heuristically some interesting avenues of research which might lead to future experimental discoveries.

One question that naturally arises at the border between GR and QM is the following: Are there novel experimental ways of studying quantized fields coupled to curved spacetime? This question has already arisen in the context of the vacuum embedded in curved spacetime [1], but I would like to extend this to possible experimental studies of the ground state of a nonrelativistic quantum many-body system with off-diagonal long-range order, i.e., a “quantum fluid,” viewed as a quantized field, coupled to curved spacetime. As we shall see, this will naturally lead to the further question: Are there *quantum* methods to detect gravitational radiation other than the *classical* ones presently being used in the Weber bar and LIGO (i.e., the “Laser Interferometer Gravitational Wave Observatory”)?

As I see it, the three main pillars of physics at the beginning of the 21st century are quantum mechanics, relativity, and statistical mechanics, which correspond to Einstein’s three papers of 1905. There exist conceptual tensions at the intersections of these three fields of physics (see Figure 1). It seems worthwhile re-examining these tensions, since they may entail important experimental consequences. In this introduction, I shall only briefly mention three conceptual tensions between these three fields: *locality* versus *nonlocality* of physical systems, *objectivity* versus *subjectivity* of probabilities in quantum and statistical mechanics (the problem of the nature of information), and *reversibility* versus *irreversibility* of time (the problem of the arrows of time). Others in this Volume will discuss the second and the third of these tensions in detail. I shall limit myself to a discussion of the first conceptual tension concerning locality versus nonlocality, mainly in the context of GR and QM. (However, in my Solvay lecture [2], I have discussed the other two tensions in more detail).

Why examine conceptual tensions? A brief answer is that they often lead to new experimental discoveries. It suffices to give just one example from late 19th and early 20th century physics: the clash between the venerable concepts of *continuity* and *discreteness*. The concept of continuity, which goes back

to the Greek philosopher Heraclitus (“everything flows”), clashed with the concept of discreteness, which goes back to Democritus (“everything is composed of atoms”). Eventually, Heraclitus’s concept of continuity, or more specifically that of the *continuum*, was embodied in the idea of *field* in the classical field theory associated with Maxwell’s equations. The atomic hypothesis of Democritus was eventually embodied in the kinetic theory of gases in statistical mechanics.

Conceptual tensions, or what Wheeler calls the “clash of ideas,” need not lead to a complete victory of one conflicting idea over the other, so as to eliminate the opposing one completely, as seemed to be the case in the 19th century, when Newton’s idea of “corpuscles of light” was apparently completely eliminated in favor of the wave theory of light. Rather, there may result a reconciliation of the two conflicting ideas, which then often leads to many fruitful experimental consequences.

Experiments on blackbody radiation in the 19th century were exploring the intersection, or borderline, between Maxwell’s theory of electromagnetism and statistical mechanics, where the conceptual tension between continuity and discreteness was most acute, and eventually led to the discovery of quantum mechanics through the work of Planck. The concept of *discreteness* metamorphosed into the concept of the *quantum*. This led in turn to the concept of *discontinuity* embodied in Bohr’s *quantum jump* hypothesis, which was necessitated by the indivisibility of the quantum. Many experiments, such as Millikan’s measurements of h/e , were in turn motivated by Einstein’s heuristic theory of the photoelectric effect based on the “light quantum” hypothesis. Newton’s idea of “corpuscles of light” metamorphosed into the concept of the *photon*. This is a striking example showing how that many fruitful experimental consequences can come out of one particular conceptual tension.

Within a broader cultural context, there have been many acute conceptual tensions between science and faith, which have lasted over many centuries. Perhaps the above examples of the fruitfulness of the resolution of conceptual tensions within physics itself may serve as a parable concerning the possibility of a peaceful reconciliation of these great cultural tensions, which may eventually lead to the further growth of both science and faith. Hence we should not shy away from conceptual tensions, but rather explore them with an honest, bold, and open spirit.

2 Three conceptual tensions between quantum mechanics and general relativity

Here I shall focus my attention on some specific conceptual tensions at the intersection between QM and GR. A commonly held viewpoint within the physics community today is that the only place where conceptual tensions between these two fields can arise is at the microscopic Planck length scale, where quantum fluctuations of spacetime (“quantum foam”) occur. Hence manifestations of

these tensions would be expected to occur only in conjunction with extremely high-energy phenomena, accessible presumably only in astrophysical settings, such as the early Big Bang.

However, I believe that this point of view is too narrow. There exist other conceptual tensions at macroscopic, non-Planckian distance scales, which should be accessible in low-energy laboratory experiments involving macroscopic QM phenomena. It should be kept in mind that QM not only describes *microscopic* phenomena, but also *macroscopic* phenomena, such as superconductivity. Specifically, I would like to point out the following three conceptual tensions:

- (1) The *spatial nonseparability* of physical systems due to entangled states in QM, versus the complete *spatial separability* of all physical systems in GR.
- (2) The *equivalence principle* of GR, versus the *uncertainty principle* of QM.
- (3) The *mixed state* (e.g., of an entangled bipartite system, one part of which falls into a black hole; the other of which flies off to infinity) in GR, versus the *pure state* of such a system in QM.

Conceptual tension (3) concerns the problem of the nature of information in QM and GR. Again, since others will discuss this tension in detail in this Volume, I shall limit myself only to a discussion of the first two of these tensions.

These conceptual tensions originate from the *superposition principle* of QM, which finds its most dramatic expression in the *entangled state* of two or more spatially separated particles of a single physical system, which in turn leads to Einstein-Podolsky-Rosen (EPR) effects. It should be emphasized here that it is necessary to consider *two or more* particles for observing EPR phenomena, since only then does the *configuration space* of these particles no longer coincide with that of ordinary spacetime. For example, consider the entangled state of two spin 1/2 particles in a Bohm singlet state initially prepared in a total spin zero state

$$|S = 0\rangle = \frac{1}{\sqrt{2}} \{ |\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \}, \quad (1)$$

in which the two particles in a spontaneous decay process fly arbitrarily far away from each other into two space-like separated regions of spacetime, where measurements on spin by means of two Stern-Gerlach apparati are performed separately on these two particles.

As a result of the quantum entanglement arising from the *superposition* of *product* states, such as in the above Bohm singlet state, it is in general impossible to factorize this state into products of probability amplitudes. Hence it is impossible to factorize the *joint* probabilities in the measurements of spin of this two-particle system. This mathematical *nonfactorizability* implies a physical *nonseparability* of the system, and leads to instantaneous, space-like correlations-at-a-distance in the joint measurements of the properties (e.g., spin) of discrete events, such as in the coincidence detection of “clicks” in Geiger

counters placed behind the two distant Stern-Gerlach apparati. These long-range correlations violate Bell's inequalities, and therefore cannot be explained on the basis of any *local realistic* theories.

Violations of Bell's inequalities have been extensively experimentally demonstrated [3]. These violations were predicted by QM. If we assume a realistic world view, i.e., that the "clicks" of the Geiger counters really happened, then we must conclude that we have observed *nonlocal* features of the world. Therefore, a fundamental *spatial nonseparability* of physical systems has been revealed by these Bell-inequalities-violating EPR correlations [4]. It should be emphasized that these space-like EPR correlations occur on macroscopic, non-Planckian distance scales, where the conceptual tension (1) between QM and GR becomes most acute.

Although some of these same issues arise in the conceptual tensions between quantum mechanics and *special* relativity, there are new issues which crop up due to the long-range nature of the gravitational force, which are absent in special relativity, but present in *general* relativity. The problem of quantum fields in *curved* spacetime can be more interesting than that in *flat* spacetime.

Gravity is a *long-range* force. It is therefore natural to expect that experimental consequences of conceptual tension (1) should manifest themselves most dramatically in the interaction of macroscopically coherent quantum matter, which exhibit *long-range* EPR correlations, with *long-range* gravitational fields. In particular, the question naturally arises: How do entangled states, such as the Bohm singlet state, interact with tidal fields, such as those in gravitational radiation [5]? Stated more generally: How do quantum many-body systems with entangled ground states possessing off-diagonal long-range order couple to curved spacetime? It is therefore natural to look to the realm of *macroscopic* phenomena associated with quantum fluids, rather than phenomena at *microscopic*, Planck length scales, in our search for these experimental consequences.

Already a decade or so before Bell's ground-breaking work on his famous inequality, Einstein himself was clearly worried by the radical, spatial nonseparability of physical systems in quantum mechanics. Einstein wrote [6]:

"Let us consider a physical system S_{12} , which consists of two part-systems S_1 and S_2 . These two part-systems may have been in a state of mutual physical interaction at an earlier time. We are, however, considering them at a time when this interaction is at an end. Let the entire system be completely described in the quantum mechanical sense by a ψ -function ψ_{12} of the coordinates q_1, \dots and q_2, \dots of the two part-systems (ψ_{12} cannot be represented as a product of the form $\psi_1\psi_2$ but only as a sum of such products [*i.e., as an entangled state*]). At time t let the two part-systems be separated from each other in space, in such a way that ψ_{12} only differs from zero when q_1, \dots belong to a limited part R_1 of space and q_2, \dots belong to a part R_2 separated from R_1

"There seems to me no doubt that those physicists who regard the descriptive methods of quantum mechanics as definitive in prin-

ciple would react to this line of thought in the following way: they would drop the requirement for *the independent existence of the physical reality present in different parts of space*; they would be justified in pointing out that the quantum theory nowhere makes explicit use of this requirement.” [*Italics mine.*]

This radical, *spatial nonseparability* of a physical system consisting of two or more entangled particles in QM, which seems to undermine the very possibility of the concept of *field* in physics, is in an obvious conceptual tension with the complete spatial separability of any physical system into its separate parts in GR, which is a *local realistic* field theory.

However, I should hasten to add immediately that the battle-tested concept of *field* has of course been extremely fruitful not only at the classical but also at the quantum level. Relativistic quantum field theories have been very well validated, at least in an approximate, correspondence-principle sense in which spacetime itself is treated classically, i.e., as being describable by a rigidly flat, Minkowskian metric, which has no possibility of any quantum dynamics. There have been tremendous successes of quantum electrodynamics and electroweak gauge field theory (and, to a lesser extent, quantum chromodynamics) in passing all known high-energy experimental tests. Thus the conceptual tension between *continuity* (used in the concept of the spacetime *continuum*) and *discreteness* (used in the concept of *quantized excitations* of a field in classical spacetime) seems to have been successfully reconciled in these relativistic quantum field theories. Nevertheless, the problem of a satisfactory relativistic treatment of quantum measurement within these theories remains an open one [7].

3 Is there any difference between the response of classical and quantum fluids to tidal gravitational fields?

Motivated by the above discussion, a more specific question arises: Is there any difference between classical and quantum matter when it is embedded in curved spacetime, for instance, in the *linear* response to the gravitational tidal field of the Earth of a *classical* liquid drop, as compared to that of a *quantum* one, such as a liquid drop of superfluid helium? In order to answer this question, consider a gedanken experiment to observe the shape of a freely floating liquid drop placed at the center of the Space Station sketched in Figure 2.

At first glance, the answer to this question would seem to be “no,” since the equivalence principle would imply that all freely falling bodies, whether classical or quantum, must respond to gravitation, e.g., Earth’s gravity, in a *mass-independent*, or more generally, a *composition-independent* way. Thus whether the internal dynamics of the particles composing the liquid drop obeys classical mechanics or quantum mechanics would seem to make no difference in the response of this body to gravity. Just as in the case of the response of the

tides of the Earth's oceans to the Moon's gravity, the shape of the surface of a liquid of any mass or composition would be determined by the equipotential surfaces of the total gravitational field, and should be independent of the mass or composition of the liquid, provided that the fluid particles can move *freely* inside the fluid, and provided that the surface tension of the liquid can be neglected.

However, one must carefully distinguish between the response of the *center of mass* of the liquid drop inside the Space Station to Earth's gravity, and the response of the *relative motions* of particles within the drop to Earth's tidal gravitational field. Whereas the former clearly obeys the mass- and composition-independence of the equivalence principle, one must examine the latter with more care. First, one must define what one means by "classical" and "quantum" bodies. By a "classical body," we shall mean here a body whose particles have undergone *decoherence* in the sense of Zurek [14], so that no macroscopic, Schrödinger-cat-like states for widely spatially separated subsystems (i.e., the fluid elements inside the classical liquid drop) can survive the rapid decoherence arising from the environment. This is true for the vast majority of bodies typically encountered in the laboratory. It is the rapid decoherence of the spatially separated subsystems of a classical body that makes the *spatial separability* of a system into its parts, and hence *locality*, a valid concept.

Nevertheless, there exist exceptions. For example, a macroscopically coherent quantum system, e.g., a quantum fluid such as the electron pairs inside a superconductor, usually possesses an energy gap which separates the ground state of the system from all possible excited states of the system. Cooper pairs of electrons in a Bardeen, Cooper, and Schrieffer (BCS) ground state are in the entangled Bohm spin singlet states given by Eq.(1). At sufficiently low temperatures, such a quantum fluid develops a macroscopic quantum coherence, as manifested by a macroscopic quantum phase which is well defined at each point inside the fluid. The resulting macroscopic wavefunction must remain single valued, in spite of perturbations, such as those due to external fields.

The energy gap, such the BCS gap, protects spatially separated, but entangled, particles within the body, such as the electrons which members of Cooper pairs inside a superconductor, against decoherence. Therefore, these quantum fluids are *protectively entangled*, in the sense that the existence of some sort of energy gap separates the nondegenerate ground state of the system from all excited states, and hence prevents any rapid decoherence due to the environment. Under these circumstances, the macroscopically entangled ground state of a quantum fluid, becomes a meaningful *global* concept, and the notion of *nonlocality*, that is, the *spatial nonseparability* of a system into its parts, enters in an intrinsic way into the problem of the interaction of matter with gravitational fields.

For example, imagine a liquid drop consisting of superfluid helium at zero Kelvin, which is in a pure quantum state, floating at the center of the Space Station, as pictured in Figure 3. Although the microscopic many-body problem for this superfluid has not been completely solved, there exist a successful macroscopic, phenomenological description based on the following Gross-Pitaevskii

equation:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(x,y,z)\Psi + \beta|\Psi|^2\Psi = -\alpha\Psi \quad (2)$$

where Ψ is the macroscopic complex order parameter, and the potential $V(x,y,z)$ describes Earth's gravity (including the tidal gravitational potential, but neglecting for the moment the frame-dragging term coupled to superfluid currents), along with the surface tension effects which enters into the determination of the free boundary of the liquid drop. Macroscopic quantum entanglement is contained in the nonlinear term $\beta|\Psi|^2\Psi$, which arises microscopically from hard-sphere atom-atom S -wave scattering events, just as in the case of the recently observed atomic Bose-Einstein condensates (BECs). (The interaction between two atoms in a individual scattering event entangles the two scattering atoms together, so that the measurement of the momentum of one atom immediately determines the momentum of the other atom which participated in the scattering event.) As in the case of the BECs, where this equation has been successfully applied to predict many observed phenomena, the physical meaning of Ψ is that it is the condensate wavefunction.

There should exist near the inside surface of the superfluid liquid drop, closed trajectories for helium atom wave packets propagating at grazing incidence, which, in the correspondence-principle limit, should lead to the atomic analog of the “whispering gallery modes” of light, such as those observed inside microspheres immersed in superfluid helium [8]. In the case of light, these modes can possess extremely high Q s (of the order of 10^9), so that the quadrupolar distortions from a spherical shape due to tidal forces can thereby be very sensitively measured optically. The atomic wave packets propagating at grazing incidence near the surface are actually those of individual helium atoms dressed by the collective excitations of the superfluid, such as phonons, rotons, and ripples [9]. Application of the Bohr-Sommerfeld quantization rule to the closed trajectories which correspond to the whispering gallery modes for atoms should lead to a *quantization* of the sizes and shapes of the superfluid drop. For a classical liquid drop, no such quantization occurs because of the decoherence of an atom after it has propagated around these polygonal closed trajectories. Hence there should exist a *difference* between classical and quantum matter in their respective responses to gravitational tidal fields.

Such a difference in the linear response between classical and matter matter in the induced quadrupole moment ΔQ_{ij} of the liquid drop can be characterized by a linear equation relating ΔQ_{ij} to the metric deviations from flat spacetime h_{kl} by means of a phenomenological susceptibility tensor $\Delta\chi_{ij}{}^{kl}$, viz.,

$$\Delta Q_{ij} = \Delta\chi_{ij}{}^{kl} h_{kl} \quad (3)$$

where i, j, k, l are spatial indices. The susceptibility tensor $\Delta\chi_{ij}{}^{kl}$ should in principle be calculable from the many-body current-current correlation function in the linear response theory of superfluid helium [10].

Here, however, I shall limit myself only to some general remarks concerning $\Delta\chi_{ij}^{kl}$ based on the Kramers-Kronig relations. Since the response of the liquid drop to weak tidal gravitational fields is *linear* and *causal*, it follows that

$$\text{Re } \Delta\chi_{ij}^{kl}(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\text{Im } \Delta\chi_{ij}^{kl}(\omega')}{\omega' - \omega} \quad (4)$$

$$\text{Im } \Delta\chi_{ij}^{kl}(\omega) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} d\omega' \frac{\text{Re } \Delta\chi_{ij}^{kl}(\omega')}{\omega' - \omega} \quad (5)$$

where P denotes Cauchy's Principal Value. From the first of these relations, there follows the zero-frequency sum rule

$$\text{Re } \Delta\chi_{ij}^{kl}(\omega \rightarrow 0) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\text{Im } \Delta\chi_{ij}^{kl}(\omega')}{\omega'}. \quad (6)$$

This relation tells us that *if* there should exist a difference in the linear response between classical and matter matter to tidal fields at DC (i.e., $\omega \rightarrow 0$) in the quadrupolar shape of the liquid drop, *then* there must also exist a difference in the rate of absorption or emission of gravitational radiation due to the imaginary part of the susceptibility $\text{Im } \Delta\chi_{ij}^{kl}(\omega')$ between classical and quantum matter. However, this argument does not tell us how big this difference is, but merely that such a difference should exist.

4 Quantum fluids versus perfect fluids

At this point, I would like to return to the more general question: Where to look for experimental consequences of conceptual tension (1)? The above discussion suggests the following answer: Look at *macroscopically entangled*, and thus *radically delocalized*, quantum states encountered, for example, in superconductors, superfluids, atomic BECs, and quantum Hall fluids, i.e., in what I shall henceforth call “quantum fluids.” Again it should be stressed that since gravity is a *long-range* force, it should be possible to perform *low-energy* experiments to probe the interaction between gravity and these kinds of quantum matter on large, non-Planckian distance scales, without the necessity of performing high-energy experiments, as is required for probing the short-range weak and strong forces on very short distance scales. The quantum many-body problem, even in its nonrelativistic limit, may lead to nontrivial interactions with weak, long-range gravitational fields, as the above example suggests. One is thereby strongly motivated to study the interaction of these quantum fluids with weak gravity, in particular, with gravitational radiation.

One manifestation of this conceptual tension is that the way one views a quantum fluid in QM is conceptually radically different from the way that one views a perfect fluid in GR, where only the *local* properties of the fluid, which can conceptually always be spatially separated into independent, infinitesimal

fluid elements, are to be considered. For example, interstellar dust particles can be thought of as being a perfect fluid in GR, provided that we can neglect all interactions between such particles [11]. At a fundamental level, the spatial separability of the perfect fluid in GR arises from the rapid decoherence of quantum superposition states (i.e., Schrödinger cat-like states) of various interstellar dust particles at widely separated spatial positions within a dust cloud, due to interactions with the environment. Hence the notion of *locality* is valid here. The response of these dust particles in the resulting *classical* many-body system to a gravitational wave passing over it, is characterized by the local, classical, free-fall motion of each individual dust particle.

In contrast to the classical case, due to their radical delocalization, particles in a macroscopically coherent quantum many-body system, i.e., a quantum fluid, are entangled with each other in such a way that there arises an unusual “quantum rigidity” of the system, closely associated with what London called “the rigidity of the macroscopic wavefunction” [12]. One example of such a rigid quantum fluid is the “incompressible quantum fluid” in both the integer and the fractional quantum Hall effects [13]. This rigidity arises from the fact that there exists an energy gap (for example, the quantum Hall gap) which separates the ground state from the low-lying excitations of the system. This gap, as pointed out above, also serves to protect the quantum entanglement present in the ground state from decoherence due to the environment, provided that the temperature of these quantum systems is sufficiently low. Thus these quantum fluids exhibit a kind of “gap-protected quantum entanglement.” Furthermore, the gap leads to an evolution in accordance with the quantum adiabatic theorem: The system stays adiabatically in a rigidly unaltered ground state, which leads in first order perturbation theory to quantum diamagnetic effects. Examples of consequences of this “rigidity of the wavefunction” are the Meissner effect in the case of superconductors, in which the magnetic field is expelled from their interiors, and the Chern-Simons effect in quantum Hall fluids, in which the photon acquires a mass inside the fluid.

5 Spontaneous symmetry breaking, off-diagonal long-range order, and superluminality

The unusual states of matter in these quantum fluids usually possess *spontaneous symmetry breaking*, in which the ground state, or the vacuum state, of the quantum many-body system breaks the symmetry present in the free energy of the system. The physical vacuum, which is in an intrinsically nonlocal ground state of relativistic quantum field theories, possesses certain similarities to the ground state of a superconductor. Weinberg has argued that in superconductivity, the spontaneous symmetry breaking process results in a broken *gauge* invariance [15], an idea which traces back to the early work of Nambu [16].

The Meissner effect in a superconductor is closely analogous to the Higgs mechanism of high-energy physics, in which the physical vacuum also spon-

taneously breaks local gauge invariance, and can also be viewed as forming a condensate which possesses a single-valued complex order parameter with a well-defined local phase. From this viewpoint, the appearance of the London penetration depth for a superconductor is analogous in an inverse manner to the appearance of a mass for a gauge boson, such as that of the W or Z boson. Thus, the photon, viewed as a gauge boson, acquires a mass inside the superconductor, such that its Compton wavelength becomes the London penetration depth. Similar considerations apply to the effect of the Chern-Simons term in the quantum Hall fluid.

Closely related to this spontaneous symmetry breaking process is the appearance of Yang's off-diagonal long-range order (ODLRO) of the reduced density matrix in the coordinate-space representation for some of these macroscopically coherent quantum systems [17]. In particular, there seems to be no limit on how far apart Cooper pairs can be inside a single superconductor before they lose their quantum coherence. ODLRO and spontaneous symmetry breaking are both purely quantum concepts with no classical analogs.

Within a superconductor there should arise both the phenomenon of instantaneous EPR correlations-at-a-distance and the phenomenon of the rigidity of the wavefunction, i.e., a Meissner-like response to radiation fields. Both phenomena involve at the microscopic level *interactions* of entangled electrons with an external environment, either through local *measurements*, such as in Bell-type experiments, or through local *perturbations*, such as those due to radiation fields interacting locally with these electrons.

Although at first sight the notion of “infinite quantum rigidity” would seem to imply infinite velocities, and hence would seem to violate relativity, there are in fact no violations of relativistic causality here, since the instantaneous EPR *correlations*-at-a-distance (as seen by an observer in the center-of-mass frame) are not instantaneous *signals*-at-a-distance, which could instantaneously connect causes to effects [18]. Also, experiments have verified the existence of superluminal wave packet propagations, i.e., faster-than- c , infinite, and even negative group velocities, for finite-bandwidth, analytic wave packets in the excitations of a wide range of physical systems [2][19]. An analytic function, e.g., a Gaussian wave packet, contains sufficient information in its early tail such that a causal medium can, during its propagation, reconstruct the entire wave packet with a superluminal pulse advancement, and with little distortion. Relativistic causality forbids only the *front* velocity, i.e., the velocity of *discontinuities* which connect causes to their effects, from exceeding the speed of light c , but does not forbid a wave packet's *group* velocity from being superluminal. One example is the observed superluminal tunneling of single-photon wave packets [20]. Thus the notion of “infinite quantum rigidity,” although counterintuitive, does not in fact violate relativistic causality.

6 The equivalence versus the uncertainty principle

Concerning conceptual tension (2), the equivalence principle is formulated at its outset using the concept of “trajectory,” or equivalently, “geodesic.” By contrast, Bohr has taught us that the very *concept* of trajectory must be abandoned at fundamental level, because of the uncertainty principle. Thus the equivalence and the uncertainty principles are in a fundamental conceptual tension. The equivalence principle is based on the idea of locality, since it requires that the region of space, inside which two trajectories of two nearby freely-falling objects of different masses, compositions, or thermodynamic states, are to be compared, go to zero volume, before the principle becomes exact. This limiting procedure is in conceptual tension with the uncertainty principle, since taking the limit of the volume of space going to zero, within which these objects are to be measured, makes their momenta infinitely uncertain. However, whenever the correspondence principle holds, the *center of mass* of a quantum wavepacket (for a single particle or for an entire quantum object) moves according to Ehrenfest’s theorem along a classical trajectory, and *then* it is possible to reconcile these two principles.

Davies [21] has come up with a simple example of a quantum violation of the equivalence principle [22][23][24]: Consider two perfectly elastic balls, e.g., one made out of rubber, and one made out of steel, bouncing against a perfectly elastic table. If we drop the two balls from the same height above the table, their classical trajectories, and hence their classical periods of oscillation will be identical, and independent of the mass or composition of the balls. This is a consequence of the equivalence principle. However, quantum mechanically, there will be the phenomenon of tunneling, in which the two balls can penetrate into the classically forbidden region *above* their turning points. The extra time spent by the balls in the classically forbidden region due to tunneling will depend on their mass (and thus on their composition). Thus there will in principle be *mass-dependent* quantum corrections of the classical periods of the bouncing motion of these balls, which will lead to quantum violations of the equivalence principle.

There exist macroscopic situations in which Ehrenfest’s correspondence principle fails. Imagine that one is inside a macroscopic quantum fluid, such as a big piece of superconductor. Even in the limit of a very large size and a very large number of particles in this object (i.e., the thermodynamic limit), there exists no correspondence-principle limit in which classical trajectories or geodesics for the *relative* motion of electrons which are members of Cooper pairs in Bohm singlet states within the superconductor, make any sense. Basically, this is due to the superposition principle and the entanglement of a macroscopic number of identical particles inside these quantum fluids. However, the motion of the center of mass of the superconductor may perfectly obey the equivalence principle, and therefore may be conceptualized in terms of a geodesic.

7 Quantum fluids as antennas for gravitational radiation

Can the quantum rigidity arising from the energy gap of a quantum fluid circumvent the problem of the tiny rigidity of classical matter, such as that of the normal metals used in Weber bars, in their feeble responses to gravitational radiation? One consequence of the tiny rigidity of classical matter is the fact that the speed of sound in a Weber bar is typically five orders of magnitude less than the speed of light. In order to transfer energy coherently from a gravitational wave by classical means, for example, by acoustical modes inside the bar to some local detector, e.g., a piezoelectric crystal glued to the middle of the bar, the length scale of the Weber bar L is limited to a distance scale on the order of the speed of sound times the period of the gravitational wave, i.e., an acoustical wavelength λ_{sound} , which is typically five orders of magnitude smaller than the gravitational radiation wavelength λ to be detected. This makes the Weber bar, which is thereby limited in its length to $L \simeq \lambda_{sound}$, much too short an antenna to couple efficiently to free space.

However, rigid quantum objects, such as a two-dimensional electron gas in a strong magnetic field which exhibits the quantum Hall effect, in what Laughlin has called an “incompressible quantum fluid” [13], are not limited by these classical considerations, but can have macroscopic quantum phase coherence on a length scale L on the same order as (or even much greater than) the gravitational radiation wavelength λ . Since the radiation efficiency of a quadrupole antenna scales as the length of the antenna L to the fourth power when $L \ll \lambda$, such quantum antennas should be much more efficient in coupling to free space than classical ones like the Weber bar by at least a factor of $(\lambda/\lambda_{sound})^4$.

Weinberg [30] gives a measure of the efficiency of coupling of a Weber bar antenna of mass M , length L , and velocity of sound v_{sound} , in terms of a branching ratio for the emission of gravitational radiation by the Weber bar, relative to the emission of heat, i.e., the ratio of the *rate* of emission of gravitational radiation Γ_{grav} relative to the *rate* of the decay of the acoustical oscillations into heat Γ_{heat} , which is given by

$$\eta \equiv \frac{\Gamma_{grav}}{\Gamma_{heat}} = \frac{64GMv_{sound}^4}{15L^2c^5\Gamma_{heat}} \simeq 3 \times 10^{-34}, \quad (7)$$

where G is Newton’s constant. The quartic power dependence of the efficiency η on the velocity of sound v_{sound} arises from the quartic dependence of the coupling efficiency to free space of a quadrupole antenna upon its length L , when $L \ll \lambda$.

Assuming for the moment that the rigidity of a quantum fluid allows us to replace the velocity of sound v_{sound} by the speed of light c (i.e., that the phase coherence of the quantum fluid allows the typical size L of a quantum antenna bar to become as large as the wavelength λ), we see that quantum fluids can be more efficient than Weber bars, based on the v_{sound}^4 factor alone, by twenty

orders of magnitude, i.e.,

$$\left(\frac{c}{v_{sound}}\right)^4 \simeq 10^{20}. \quad (8)$$

Thus quantum fluids could be much more efficient receivers of this radiation than Weber bars for detecting astrophysical sources of gravitational radiation. This has previously been suggested to be the case for superfluids and superconductors [25][26][27].

Another important property of quantum fluids is the fact that they can possess an extremely low dissipation coefficient Γ_{heat} , as can be inferred, for example, by the existence of persistent currents in superfluids that can last for indefinitely long periods of time. Thus the impedance matching of the antenna to free space [28], or equivalently, the branching ratio of energy emitted into the gravitational radiation channel rather than into the heat channel, can be much larger than that calculated above for the classical Weber bar.

8 Minimal-coupling rule for a quantum Hall fluid

The electron, which possesses charge e , rest mass m , and spin $s = 1/2$, obeys the Dirac equation. A nonrelativistic interacting many-electron system, such as that in the quantum Hall fluid, should obey the minimal-coupling rule which originates from the covariant-derivative coupling of the Dirac electron to curved spacetime [1], viz.,

$$p_\mu \rightarrow p_\mu - eA_\mu - \Sigma_{AB}\Gamma_\mu^{AB} \quad (9)$$

where p_μ is the electron's four-momentum, A_μ is the electromagnetic four-potential, Σ_{AB} are the Dirac γ matrices in curved spacetime with tetrad (or vierbein) A, B indices, and Γ_μ^{AB} are the components of the spin connection. In the nonrelativistic limit, the four-component Dirac spinor is reduced to a two-component spinor. While the precise form of the nonrelativistic Hamiltonian is not known for the many-body system in a weakly curved spacetime consisting of electrons in a strong magnetic field, I conjecture that it will have the form

$$H = \frac{1}{2m} (p_i - eA_i - s_{ab}c_i^{ab})^2 + V \quad (10)$$

where i is a spatial index, a, b are spatial tetrad indices, s_{ab} is a two-by-two matrix-valued tensor representing the spin, and $s_{ab}c_i^{ab}$ is the nonrelativistic form of $\Sigma_{AB}\Gamma_\mu^{AB}$. Here H and V are two-by-two matrix operators on the two-component spinor electron wavefunction in the nonrelativistic limit. The potential energy V includes the Coulomb interactions between the electrons in the quantum Hall fluid. This nonrelativistic Hamiltonian has the form

$$H = \frac{1}{2m} (\mathbf{p} - \mathbf{a} - \mathbf{b})^2 + V, \quad (11)$$

where the particle index, the spin, and the tetrad indices have all been suppressed [29]. Upon expanding the square, it follows that for a quantum Hall fluid of uniform density, there exists a cross-coupling or interaction Hamiltonian term of the form

$$H_{int} \sim \mathbf{a} \cdot \mathbf{b} \quad (12)$$

which couples the electromagnetic \mathbf{a} field to the gravitational \mathbf{b} field. In the case of time-varying fields, $\mathbf{a}(t)$ and $\mathbf{b}(t)$ represent EM and GR radiation, respectively.

The question immediately arises: EM radiation is fundamentally a spin 1 (photon) field, but GR radiation is fundamentally a spin 2 (graviton) field. How is it possible to convert one kind of radiation into the other, and not violate the conservation of angular momentum? The answer: The EM wave converts to the GR wave *through a medium*. Here specifically, the medium of conversion consists of a strong DC magnetic field applied to a system of electrons. This system possesses an axis of symmetry pointing along the magnetic field direction, and therefore transforms like a spin 1 object. When coupled to a spin 1 (circularly polarized) EM radiation field, the total system can in principle produce a spin 2 (circularly polarized) GR radiation field, by the addition of angular momentum. However, it remains an open question as to how strong this interconversion process is between EM and GR radiation. Most importantly, the size of the conversion efficiency of this transduction process needs to be determined by experiment.

We can see more clearly the physical significance of the above interaction Hamiltonian H_{int} once we convert it into second quantized form and express it in terms of the creation and annihilation operators for the positive frequency parts of the two kinds of radiation fields, as in the theory of quantum optics, so that in the rotating-wave approximation

$$H_{int} \sim a^\dagger b + b^\dagger a \quad (13)$$

where the annihilation operator a and the creation operator a^\dagger of the single classical mode of the plane-wave EM radiation field corresponding to the \mathbf{a} term, obey the commutation relation $[a, a^\dagger] = 1$, and where the annihilation operator b and the creation operator b^\dagger of the single classical mode of the plane-wave GR radiation field corresponding to the \mathbf{b} term, obey the commutation relation $[b, b^\dagger] = 1$. (This represents a crude, first attempt at quantizing the gravitational field, which applies only in the case of weak, linearized gravity.) The first term $a^\dagger b$ then corresponds to the process in which a graviton is annihilated and a photon is created inside the quantum fluid, and similarly the second term $b^\dagger a$ corresponds to the reciprocal process, in which a photon is annihilated and a graviton is created inside the quantum fluid.

A Berry phase picture [31] of a spin coupled to curved spacetime leads to an intuitive way of understanding why there could exist a coupling between a GR wave and an EM wave mediated by the quantum Hall fluid. The spin of the electron is always constrained to lie on the space-like submanifold of

curved spacetime, which is perpendicular to the local time axis. This is due to the fact that we can always transform to a co-moving frame, such that the electron is at rest at the origin of this frame. In this frame, the spin of the electron must be purely a space-like vector with no time-like component. This imposes an important *constraint* on the motion of the electron's spin, such that whenever the space-like submanifold of spacetime is disturbed by the passage of a gravitational wave, the spin must remain at all times *perpendicular* to the local time axis. If the spin vector is constrained to follow a conical trajectory during the passage of the gravitational wave, the electron will pick up a Berry phase proportional to the solid angle subtended by this conical trajectory after one period of the GR wave. Such a Berry phase can induce a macroscopic, coherent electrical current in the quantum Hall fluid, in a manner similar to the persistent currents induced by the Berry phase in ODLRO systems [32]. The macroscopic electrical current can generate an EM wave. In this manner, a GR wave can be converted into an EM wave. By reciprocity, the time-reversed process of the conversion from an EM wave to a GR wave must also be possible.

There exists other situations in which a minimal-coupling rule similar to the one above, arises for *scalar* quantum fields in curved spacetime. DeWitt [33] suggested in 1966 such a coupling in the case of superconductors [34]. Speliotopoulos [35] noticed in 1995 that a cross-coupling term of the form of Eq.(12) arose in the long-wavelength limit of a certain quantum Hamiltonian derived from the geodesic deviation equations of motion using the transverse-traceless gauge. Speliotopoulos and I are currently working on the problem of the minimal-coupling rule for a scalar quantum field coupled to curved spacetime in a generalized laboratory frame, which avoids the use of the long-wavelength approximation.

For quantum fluids which possess a macroscopic complex order parameter Ψ obeying the Ginzburg-Landau equation, the above minimal-coupling rule suggests that this equation be generalized as follows:

$$\frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \mathbf{a} - \mathbf{b} \right)^2 \Psi + \beta |\Psi|^2 \Psi = -\alpha \Psi. \quad (14)$$

9 Quantum transducers between EM and GR waves?

The above discussion suggests that there might exist quantum transducers between EM and GR waves based on the cross-coupling Hamiltonian $H_{int} \sim \mathbf{a} \cdot \mathbf{b}$. One possible geometry for an experiment is shown in Figure 4. An EM wave impinges on the quantum fluid, such as the quantum Hall fluid described above, which converts it into a GR wave in process (a). In the time-reversed process (b), a GR wave impinges on the quantum fluid, which converts it back into an EM wave. It is an open question at this point as to what the conversion efficiency of such quantum transducers will be. This question is best settled by an experiment to measure this efficiency by means of a Hertz-like apparatus, in

which process (a) is used for generating gravitational radiation, and process (b) is used to detect this radiation. If the quantum transducer efficiency turns out to be high, this will lead to an avenue of research which could be called “gravity radio.” I have performed a preliminary version of this Hertz-like experiment with Walt Fitelson using the high T_c superconductor YBCO to measure its transducer efficiency at microwave frequencies. Results of this experiment will be reported elsewhere.

10 Conclusions

The conceptual tensions between QM and GR, the two main fields of interest of John Archibald Wheeler, could indeed lead to important experimental consequences, much like the conceptual tensions of the past. I have covered here in detail only one of these conceptual tensions, namely, the tension between the concept of *spatial nonseparability* of physical systems due to the notion of non-locality embedded in the superposition principle, in particular, in the entangled states of QM, and the concept of *spatial separability* of all physical systems due to the notion of locality embedded in the equivalence principle in GR. This has led to the idea of antennas and transducers using quantum fluids, such as the quantum Hall fluid, as potentially practical devices, which could possibly open up a door for further exciting discoveries.

11 Acknowledgments

I dedicate this paper to my teacher, John Archibald Wheeler, whose vision helped inspire this paper. I am grateful to the John Templeton Foundation for the invitation to contribute to this Volume, and would like to thank my father-in-law, the late Yi-Fan Chiao, for his financial and moral support of this work. This work was supported also by the ONR.

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12 Figure Captions

1. Figure 1: *Three intersecting circles* in a Venn-like diagram represent the three main pillars of physics at the beginning of the 21st century. The top circle represents quantum mechanics, and is labeled by Planck's constant \hbar . The left circle represents relativity, and is labeled by the two constants c , the speed of light, and G , Newton's constant. The right circle represents statistical mechanics and thermodynamics, and is labeled by Boltzmann's constant k_B . Conceptual tensions exist at the intersections of these three circles, which may lead to fruitful experimental consequences.
2. Figure 2: *Liquid drop* placed at the center of a not-to-scale sketch of the Space Station, where it is subjected to the tidal force due to the Earth's gravity. Is there any difference between the shape of a classical and a quantum liquid drop, for example, between a drop of water and one composed of superfluid helium?
3. Figure 3: *Whispering gallery modes* of a liquid drop arise in the correspondence principle limit, when an atom or a photon wave packet bounces at grazing incidence off the inner surface of the drop in multiple specular internal reflections, and forms a closed polygonal trajectory. The Bohr-Sommerfeld quantization rule leads to a discrete set of such modes.
4. Figure 4: *Transducer* between electromagnetic (EM) and gravitational (GR) radiation, consisting of a quantum fluid with charge and spin, such as the quantum Hall fluid. The minimal-coupling rule for an electron with its spin coupled covariantly to curved spacetime, results in two processes: In process (a) an EM plane wave is converted upon reflection from the quantum fluid into a GR plane wave; in process (b), which is the reciprocal or time-reversed process, a GR plane wave is converted upon reflection from the quantum fluid into an EM plane wave. Transducer interconversion between these two kinds of waves may also occur upon *transmission* through the quantum fluid, as well as upon *reflection*.

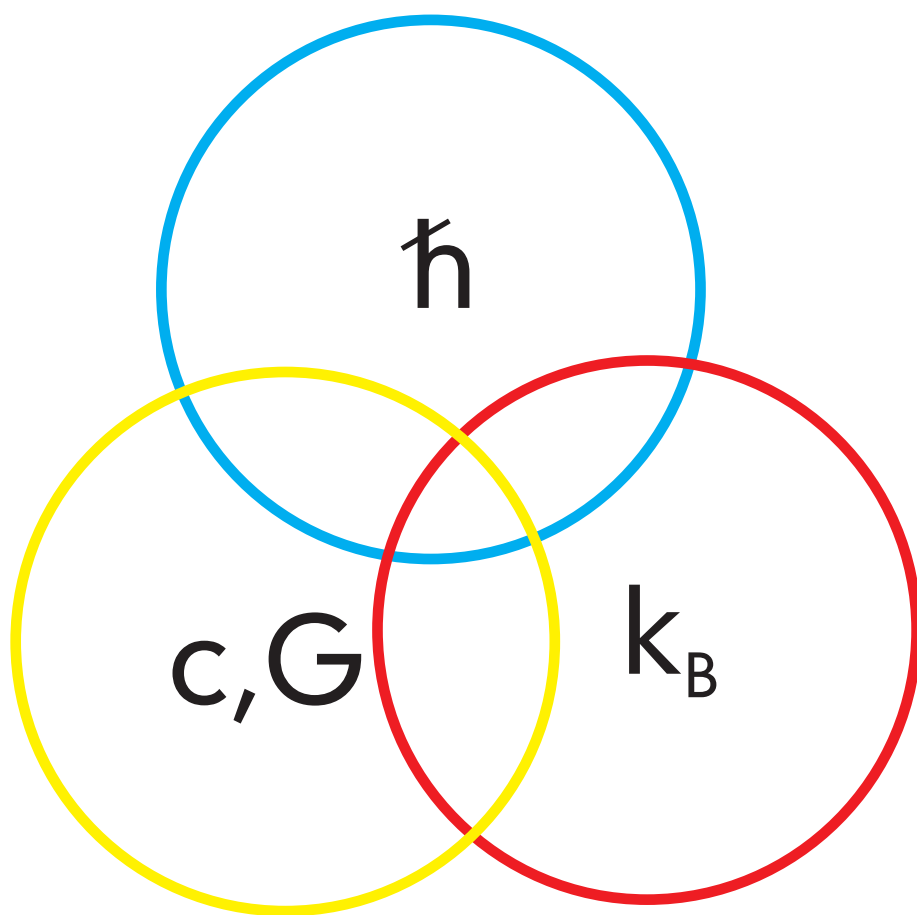


Figure 1:

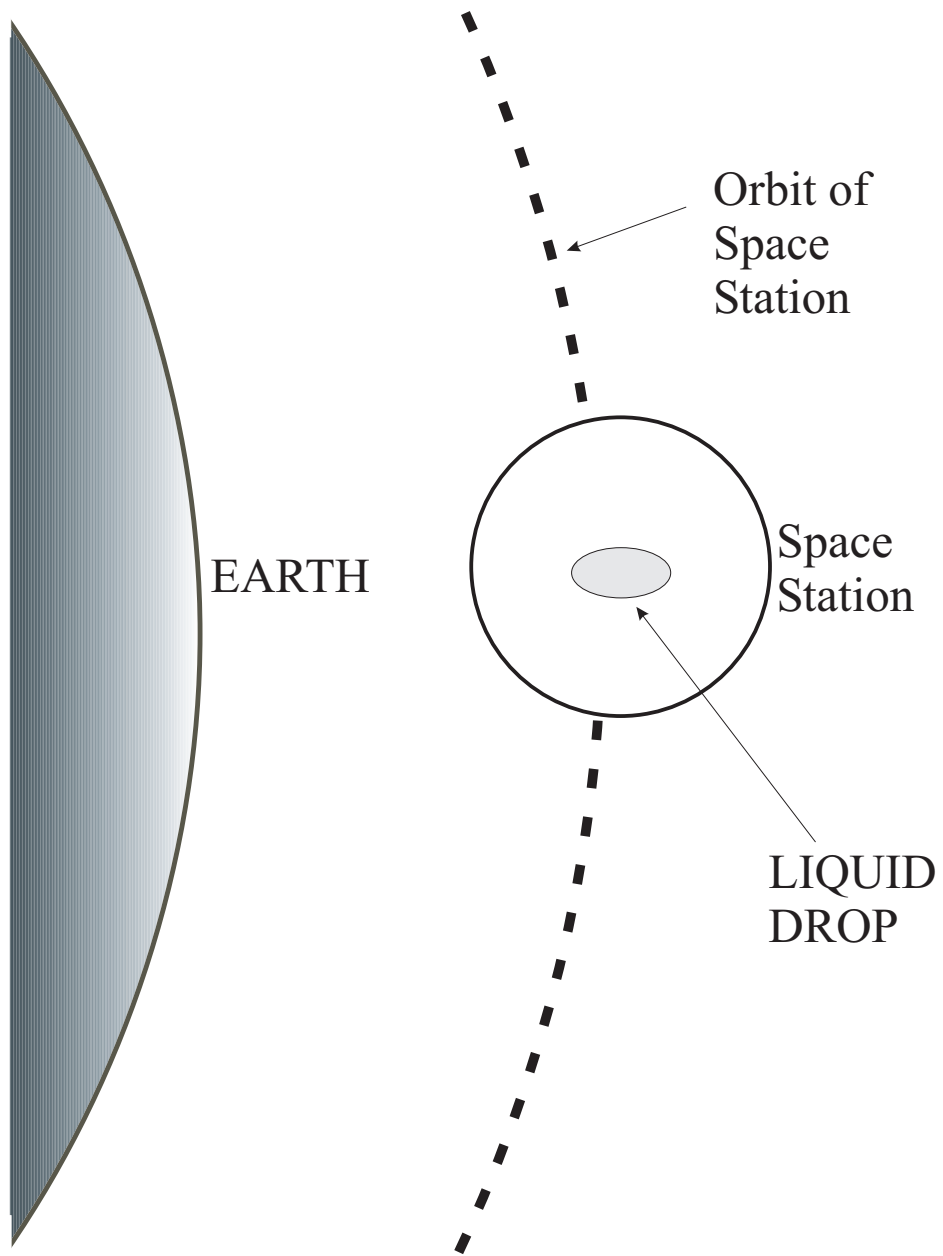
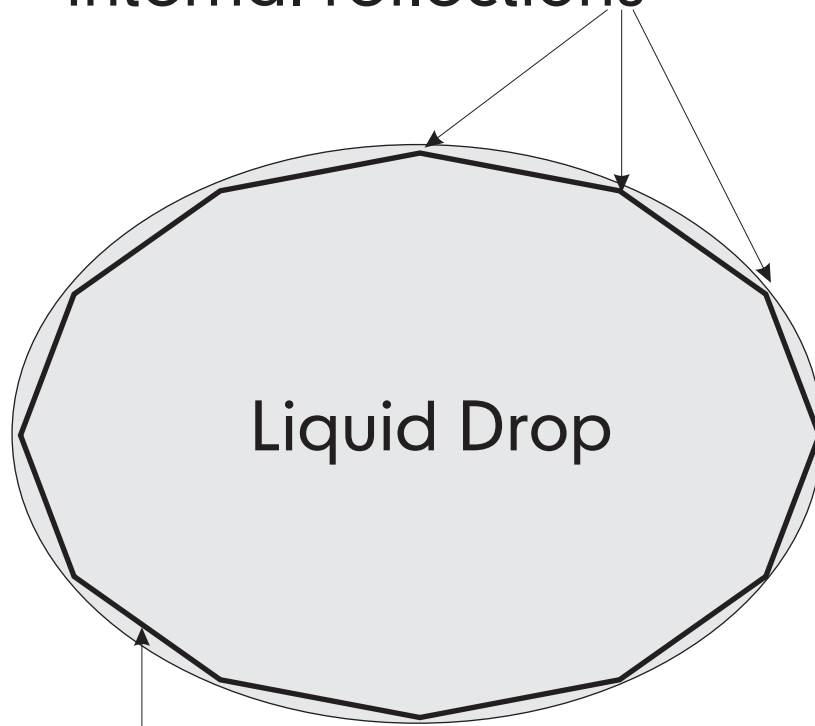


Figure 2:

Multiple specular total
internal reflections



Liquid Drop

Closed trajectory of an
atom or photon wave
packet

Figure 3:

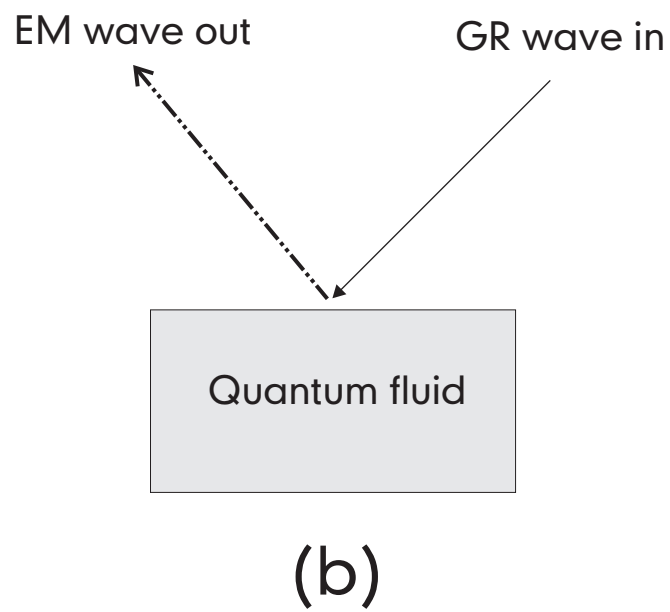
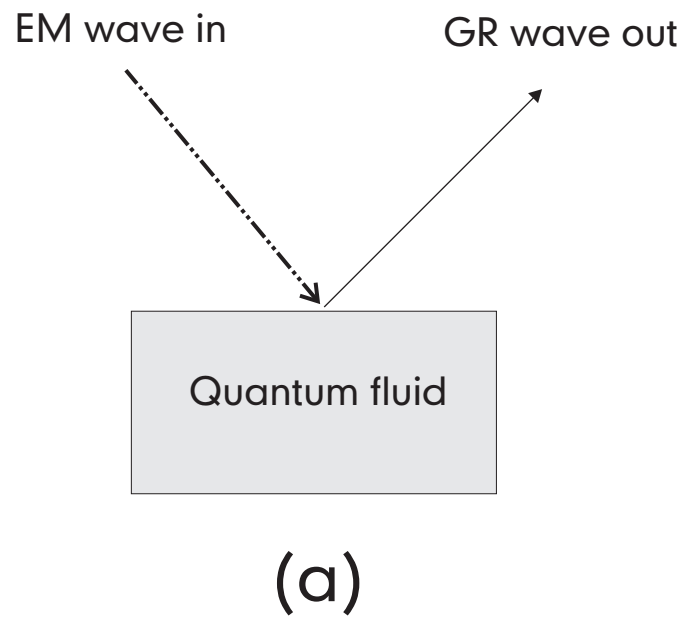


Figure 4: